Tunable negative refraction based on quantum interference

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Model

normal refraction
(Snell’s law)

\[
\theta' < \theta
\]
Model

\[
n = 1 \quad \text{and} \quad n' = -1
\]

Veselago (68), Pendry (90’s)

negative refraction (Snell’s law) \quad \text{“left-handed”: } E, B, \text{ and } k
normal refraction

negative refraction
Contents

• Definitions & examples
• Chiral media
• EIT based negative refraction
  – Local field effects
  – Tunability
• Conclusion
Origin of negative refraction

\[ n = \sqrt{\varepsilon \mu} \]

\( \varepsilon \): electric permittivity

\( \mu \): magnetic permeability

With both, \( \varepsilon \) and \( \mu \) negative \( \Box \) \( n \) negative
Applications and Definitions

• Applications:
  – perfect lens
Superlens

normal lens:

\[ k_x = \sqrt{\frac{\omega^2}{c^2} - k_z^2} \]

☑ Resolution: \[ 2\pi k_x^{-1} \geq \lambda \]
Applications and Definitions

- **Applications:**
  - perfect lens

- **Definitions:**
  - negative refraction
  - left-handed materials
  - chiral materials
  - meta-materials

Re(n) < 0

\(E, B, \text{ and } k\) are lefthanded

Turned polarization \(\leftrightarrow E\)-, \(B\)-cross coupled

man-made refraction
Material examples

• μ-wave structures:

Pendry

Shalaev
Photonic bandgap material

- Use band structure of the photonic crystal to get a left-handed material ("flip over" $k$ vector direction on Fermi surface)

- For certain frequency: negative refraction
- But: not "metamaterial": No resolution beyond $\lambda$! (✓ no superlensing!)
Absorption

So far: refraction/absorption ≈ 1…5

Our case: Re(n)/Im(n) = 100
Occurrence of negative refraction

- Why does negative index not occur in Nature?

Large $\chi_m$ very difficult to achieve!
Optical frequencies

Magnitude of $\chi_m$:

$$|\chi_m| \approx \left( \frac{\mu_{\text{atom}}}{d_{\text{atom}}} \right)^2 |\chi_e| \approx \frac{1}{137^2} |\chi_e|$$
Chiral media (Pendry)

• Remember: \[ n = \sqrt{\varepsilon \mu} \]
Chiral media (Pendry)

- Remember: \[ n = \sqrt{\varepsilon \mu} \]
- Chiral media: cross coupling between electric and magnetic fields
  \[ P = \chi_e E + \xi_{eb} B \]
  \[ M = \xi_{be} E + \chi_m B \]
  with \[ |\xi| \propto \frac{1}{137} |\chi_e| \]

- Index of refraction
  \[ n = \sqrt{\varepsilon \mu - \xi} \]

If we choose \[ \xi_{EH} = -\xi_{HE} = i\xi \]
EIT based negative refraction

V-type system:
- \( E, B \) electric/magnetic part of probe field
- \( \Omega \) cross couples electric and magnetic transition

Chiral behavior
- \( \gamma_0 \ll \gamma \), EIT

\[ |1 \rangle_g \rightarrow |2 \rangle_g \rightarrow |3 \rangle_g \]

absorption \((\chi_e'')\)
dispersion \((\chi_e')\)
EIT based negative refraction

Problems:

- $\Omega$: dc-coupling $\checkmark$, phase of $\xi$ not free to choose
- $\Omega$ dc-coupling: very weak Rabi frequency
- no EIT for inhomogeneously broadened systems
- level scheme hard to find in real systems
Realistic schemes

- Create dark state in superposition of $|1\rangle$ and $|4\rangle$

- Dark state acts like g.s. in 3-level system

\[ |\text{dark}\rangle \propto \Omega_1 |1\rangle - \Omega_2 |4\rangle \]
Realistic schemes

Advantages:
• Non-dc coupling field $\Omega$

Choose phase
Realistic schemes

Advantages:

- Non-dc coupling field $\Omega$
  - Choose phase
- States $|2\uparrow\rangle$ and $|4\uparrow\rangle$ can be chosen at similar energy
  - No Doppler broadening on sensitive $\Lambda$-type scheme ($|4\uparrow\rangle$, $|2\uparrow\rangle$, and $|3\uparrow\rangle$)
- Easier to realize

- $|3\uparrow\rangle$
- $|2\uparrow\rangle$
- $|5\uparrow\rangle$
- $|E\uparrow\rangle$
- $|B\uparrow\rangle$
- $|4\uparrow\rangle$
Realistic schemes

\[ \gamma = \gamma_3 \]

\[ \gamma_2 \]

\[ \gamma_5 \]

\[ + \text{ line broadening (inhomogeneous)} \]
Cross couplings

Inhomogeneous broadening $\approx$ decay rate $\gamma$

$\chi_{e}$
$\xi_{eb}$
$\xi_{be}$
$\chi_{m}$

real part
imaginary part
Index of refraction

density $N = 5 \times 10^{16}$ cm$^{-3}$

$$n$$

real part

imaginary part

detuning from resonance $\Delta/\gamma$
Local field corrections

\[ E \rightarrow E_{loc} = E + \frac{4\pi}{3} P \]
\[ B \rightarrow B_{loc} = B + \frac{4\pi}{3} M \]

re-calculate \( \chi \)'s and \( \xi \)'s . . .
Local field corrections

density $N = 5 \times 10^{16} \text{ cm}^{-3}$

detuning from resonance $\Delta/\gamma$
Density dependence

imaginary part \(\rightarrow 100\)

real part

Logarithm of density \(10^x\) cm\(^{-3}\)
**Fine tuning**

$n$ can be **tuned** by changing coupling field Rabi frequency $\Omega$:

![Graph showing the real and imaginary parts of $n$ as a function of the logarithm of Rabi frequency $\Omega = 10^x \gamma$.](image)

Application: e.g., for superlens, $n=-1$ is needed **exactly**!
Realization schemes

• **Atoms**: e.g. Neon

• **Molecules**: Use different rotational levels for different parities

• **Bound excitons**: use $D^0$ states with different parities for lower, and $D^0X$ states with different parities for upper states.
Outlook

• Materials:
  – Problem of high-frequency M1 transitions in atoms and molecules
  – Parity in solid state systems
• Dimension: 3D?
• Comparison with “traditional” method + gain
• Systems:
  – Optimize level scheme
  – Utilize tensorial character of $\varepsilon$
Conclusions

• Use of negative refraction:
  – superlenses and others

• Metamaterials:
  – chiral media for presence of cross coupling
  – EIT for suppression of absorption
  – energy and Rabi freq. of coupling fields for tuning
<table>
<thead>
<tr>
<th></th>
<th>normal refraction</th>
<th>negative refraction</th>
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<tbody>
<tr>
<td>phase velocity $v \approx c$</td>
<td>Group velocity $v_{gr} &lt; v$</td>
<td>phase velocity $v \approx -c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>group velocity $v_{gr} \approx +c$</td>
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</tbody>
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Problem: absorption

Kramers - Kronig: relationship between refraction/absorption

large $\chi_e'$ (refraction) □ large $\chi_e''$ (absorption)
Cross couplings

atomic picture:

\[
\rho_{34} = \alpha_{ee} E + \alpha_{eb} B
\]

\[
\rho_{21} = \alpha_{be} E + \alpha_{bb} B
\]

Solve for \( \alpha \) \( \checkmark \) . . .
Different approach

• usual problem: $\mu (\chi_m)$
• Instead: leave $\mu$ and make $\varepsilon$ into tensor (“geometric approach”)

normal
birefringent
“very birefringent”

$\vec{k}$ wave vector

$\vec{S}$ Poynting vector

Disadvantage: works only in waveguide (i.e. 1D)

Podolskiy, Narimanov, PRB R201101, (2005)
Neon

2p^6

2p^5(2P_{3/2})3s

352 nm

2p^5(2P_{1/2})3s

5.4 μm

2p^5(2P_{1/2})4p

2p^5(2P_{1/2})3d

Thommen, Mandel, PRL 96, 053601 (2006)
Molecular or solid state levels

one even, one odd parity (e.g., even and odd rotational level) for \(|1\text{g}\) and \(|4\text{g}\)
Bound exciton

momentum picture:

singlet

exciton

quadruplet

cited state ($D^0X$)

ground state ($D^0$)

(e.g., 5-electron atom in a 4-valence electron lattice)
Bound exciton

momentum picture:

singlet

donor

exciton

quadruplet

vb

cb