Dissociation dynamics of Bose-Fermi mixtures in lattices

Takahiko Miyakawa and PM
Dept. of Physics & College of Optical Sciences
The University of Arizona
The model

$t=0$:
Molecules only, either MI or SF phase

$t>0$:
Photoassociation/dissociation laser switched on

Collisions, tunneling, Mott insulator vs. superfluid, ...
Spin-Boson lattice Hamiltonian

\[ H_{BF} = H_f + H_b + V_{bf} \]

- Fermionic atoms:
  \[ H_f = \hbar \omega_f \sum_{i,\sigma} n_{i\sigma}^f - \hbar J_f \sum_{\langle ij \rangle,\sigma} (f_{i\sigma}^{\dagger} f_{j\sigma} + H.c.) + \hbar U_f \sum_i n_{i\uparrow}^f n_{i\downarrow}^f \]

- Bosonic molecules:
  \[ H_b = \hbar (\omega_d + \omega_b) \sum_i n_i^b - \hbar J_b \sum_{\langle ij \rangle} (b_i^{\dagger} b_j + H.c.) + \frac{1}{2} \hbar U_b \sum_i n_i^b (n_i^b - 1) \]

- Photoassociation:
  \[ V_{bf} = \hbar g \sum_i (f_{i\uparrow}^{\dagger} f_{i\downarrow}^{\dagger} b_i + H.c.) + \hbar U_{bf} \sum_{i\sigma} n_{i\sigma}^f n_i^b \]

- tunneling
- collisions
- photoassociation
Fermions: Strongly confined regime

- Treat tunneling perturbatively
- But: fermionic pairs created at same site by photodissociation

→ Eliminate unpaired states adiabatically
Spin-boson Hamiltonian

Pseudo-spin representation:

\[ s_i^+ = f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger \]
\[ s_i^- = f_{i\downarrow} f_{i\uparrow} \]
\[ s_i^z = \frac{1}{2}(n_{i\uparrow}^f + n_{i\downarrow}^f - 1) \]

Spin-boson Hamiltonian!

\[ H_f \rightarrow H_s = \hbar \omega_s \sum_i (2s_i^z + 1) + \hbar J_s \sum_{\langle ij \rangle} (s_i^x s_j^x + s_i^y s_j^y - s_i^z s_j^z) \]

\[ V_{bf} \rightarrow V_{bs} = \hbar U_{bf} \sum_i (2s_i^z + 1) + \hbar g \sum_i (b_i^\dagger s_i^- + H.c.) \]

\[ \omega_s = \omega_f + U_f / 2 \]

\[ J_s = 4J_f^2 / U_f \]
Gutzwiller ansatz

\[ |\Psi(t)\rangle = \prod_{i=1}^{N_x} \left( \sum_{n_i=0}^{\infty} \sum_{\sigma_i=-1/2}^{1/2} f_{n_i,\sigma_i}^{(i)}(t) |n_i,\sigma_i\rangle \right) \]

- \( \sigma_i \): Fermi pair occupation of site \( i \)
- \( n_i \): Bosonic occupation of site \( i \)

- Time-dependent variational principle:

\[ \sum_{n_i,\sigma_i} \left| f_{n_i,\sigma_i}^{(i)}(t) \right|^2 = 1 \]

\[ \frac{\partial}{\partial f_{n_i,\sigma_i}^{(i)*}} \left\langle \Psi(t) \left| i\hbar \frac{\partial}{\partial t} - H_{SB} \right| \Psi(t) \right\rangle = 0 \]
Initial molecular ground state
(No photoassociation)

- Superfluid: \( U_b / zJ_b \leq 1 \)
  \[
f_{n\downarrow} = e^{-\nu/2} \frac{\left(\sqrt{\nu}\right)^n}{\sqrt{n!}}
\]

- Mott insulator: \( U_b / zJ_b \geq 1 \)
  \[
f_{n\downarrow} = \delta_{n,\nu}
\]
Generalized Jaynes-Cummings dynamics

\[ J_b, J_s \to 0 \]

\[ H = \hbar (\omega_d + \omega_b) \sum_i n_i^b + \sum_i \hbar \omega_s (2s_{fi}^z + 1) + \hbar g \sum_i (b_i^+ s_{fi}^- + b_i s_{fi}^+) + \frac{1}{2} \hbar U_b \sum_i n_i^b (n_i^b - 1) \]

MI phase

superfluid

Gutzwiller dynamics

\[ zJ_b / g = 0.1 \]
Generalized Tavis-Cummings dynamics

- Extreme superfluid regime
- Single-mode approximation for molecular field

\[ zJ_b \gtrsim g \]
\[ U_b / zJ_b \lesssim 1 \]

\[ H \rightarrow \hbar \delta_0 n_0 + \frac{\hbar g}{\sqrt{N_s}} \left[ b_0 S_f^- + b_0^\dagger S_f^+ \right] + \frac{\hbar U_0}{2N_s} n_0^2 \]

\[ S_f^+ = \sum_i S_{fi}^+ \quad \text{Collective spin operator} \]

\[ \delta_0 = \delta - zJ_b - 2\nu U_{bf} \]
\[ U_0 = U_b - 4U_{bf} \]

Collective oscillations between atoms and molecules
Classical limit

\[ n_0 \approx \langle n_0 \rangle \rightarrow \frac{d^2}{dt^2} \langle n_0 \rangle = -\frac{d}{d\langle n_0 \rangle} V(\langle n_0 \rangle) \]

\[
V(\langle n_0 \rangle) = \frac{U_0^2}{8N_s^2} \langle n_0 \rangle^4 + (\cdots) \langle n_0 \rangle^3 + (\cdots) \langle n_0 \rangle^2 + (\cdots) \langle n_0 \rangle ; \quad U_0 = U_e - 4U_{bf}
\]
Gutzwiller mean-field dynamics

\[ |\Psi(t)\rangle = \prod_{i=1}^{N} \left( \sum_{n_i=0}^{\infty} \sum_{\sigma_i=-1/2}^{1/2} f_{n_i,\sigma_i}^{(i)}(t) |n_i,\sigma_i\rangle \right) \]

\[
i \frac{\partial f_{n,\sigma}}{\partial t} = h_{n,\sigma} f_{n,\sigma} + g \left[ \sqrt{n} f_{n-1,\sigma+1} + \sqrt{n+1} f_{n+1,\sigma-1} \right] - z J_b \left[ \Phi^* \sqrt{n+1} f_{n+1,\sigma} + \Phi \sqrt{n} f_{n-1,\sigma} \right] + (z/2) J_s \left[ \Delta^* f_{n,\sigma+1} + \Delta f_{n,\sigma-1} - 2\sigma M f_{n,\sigma} \right]
\]

\[ \Phi(t) = \langle b(t) \rangle \quad \text{bosonic order parameter} \]

\[ \Delta(t) = \langle s_f^-(t) \rangle \quad \text{fermionic pairs order parameter} \]

\[ M(t) = \langle s_f^z(t) \rangle \quad \text{pseudo-spin magnetization} \]
Gutzwiller dynamics \((J_s = 0)\)

Separation between superfluid and MI initial conditions
Gutzwiller dynamics

\[ zJ_b / g = 0.1 \]

\[ zJ_b / g = 1 \]

\[ zJ_b / g = 10 \]
Gutzwiller probabilities

Remember: 
\[ |\Psi(t)\rangle = \prod_{i=1}^{N} \left( \sum_{\substack{n_i=0 \sigma_i=-1/2}}^{\infty} \sum_{\substack{n_i=0 \sigma_i=+1/2}}^{1/2} f_{n_i,\sigma_i}(t) |n_i,\sigma_i\rangle \right) \]
Self-trapping transitions

\[ U_b / zJ_b = 1.50 \quad U_b / zJ_b = 1.86 \quad U_b / zJ_b = 1.87 \]

“coherent” self-trapping transition

\[ U_b / zJ_b = 4.70 \quad U_b / zJ_b = 4.71 \quad U_b / zJ_b = 5.50 \]

“incoherent” self-trapping transition
Self-trapping solutions

Physical origin: quartic term in classical potential

\[ V(\langle n_0 \rangle) = \frac{U_0^2}{8N_s} \langle n_0 \rangle^4 + \cdots \langle n_0 \rangle^3 + \cdots \langle n_0 \rangle^2 + \cdots \langle n_0 \rangle \quad ; \quad U_0 = U_b - 4U_{bf} \]
Summary and outlook

- Weak molecular tunneling regime: Jaynes-Cummings dynamics
- Strong molecular tunneling regime: Tavis Cummings dynamics
- Coherent and incoherent self-trapping transitions as function of $U_b / zJ_b$

NEXT:
- XXZ terms (intersite spin-spin coupling)
- Strong fermionic tunneling
- ...