Formation of Vortices and Vortex Clusters from Matter-wave Interference in BECs

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ARO         NSF
To understand how the role of **defects and roughness** in a confining potential affects the formation and dynamics of BECs, and superfluids in general.

**Main Objective**

Interference, Turbulence, Vortices
Motivation and Importance

• Condensed-matter physics: better understanding of the establishment of superconducting, superfluid states. BEC is a model system, easier to study “dirty” effects (i.e., defects) by starting with a “clean” system such as achieved with BECs.

• Applications: BEC transport in waveguides, atom-optical elements (ex: beam-combiners)

• New topics in BEC: Characterizing superfluids in disordered potentials

• Fundamental issues: fragmentation, onset of condensation, symmetry-breaking transitions. “Can a BEC form in an excited state of a potential?”

• Quantum (non)-control: Means for (possibly undesired) excitations to enter system: vortex formation and pinning. BEC dynamics based on indeterminate initial conditions.

• Matter-wave interference: fundamental link to turbulence, generation of vortices, vortex clusters.

• Quantum-state engineering: possible new methods of state preparation

How do quantum fluids merge together?
**Experiment Idea**

1. Approximate a “rough” potential with a **weak** bump in the middle of a harmonic oscillator potential.

- Negligible effect on thermal cloud
- Just one BEC in the overall potential at the end of evaporative cooling

2. Make a BEC in this new potential well.

*Top-down view:*

“Seed” BECs start to form in 3 places, **but they merge together as they grow**

*A trap with 3 wells*
Approach, & Talk Outline

Theory

1. Concept: Build an understanding of how the mixing of quantum fluids may produce vortices.

2. Numerics: Use GPE to model the growth of a (quasi-2D) BEC in a bumpy potential

Experiment

3. Optical barrier: Shape a blue-detuned laser beam, add to a weak TOP trap to make bumpy trap

4. Condense: Create $^{87}$Rb BECs in the bumpy trap

5. Measurements: Look for vortices
Concept: Growth of a BEC

(1) 3 independent seed BECs start forming from common thermal cloud.

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Barrier energy $E_B$

Single-particle ground-state energy $E_0 \ll E_B$

(2) Seed BECs grow, merge, establish relative phases (random), first by tunneling, then above-barrier transport.

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Chemical potential $\mu < E_B$

(3) BECs merge together into one final BEC. Interference leads to fluid flow, excitations.

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Final chemical potential $\mu > E_B$

Interference region

Fluid flow? Depends on phase gradients and relative phases.

Interference region

Fluid flow? Depends on phase gradients and relative phases.
Fluid flow over barriers

Assume a symmetric superposition state with variable phases $\phi_1$ and $\phi_2$

$$\Psi = \sqrt{n_1(x + x_0)}e^{i\phi_1} + \sqrt{n_2(x - x_0)}e^{i\phi_2}$$

Current density:

$$J(x) = \frac{\hbar}{2im} \left[ \Psi^* \frac{\partial \Psi}{\partial x} - \frac{\partial \Psi^*}{\partial x} \Psi \right]$$

$$J(x = 0) = \frac{\hbar n_2(x = 0)}{m} \sin(\phi_2 - \phi_1)$$  \hspace{1cm} (assumes $n_1 = n_2$, $dn_1/ dt = -dn_2/ dt$)

Mass current (fluid flow) direction depends on phase difference:
true for Josephson Effect and above-barrier transport.

Neglects phase gradients, potentially very important, may lead to interference fringes in the growing BEC.
A Vortex from matter-wave interference

\[ J \sim \sin(\phi_2 - \phi_1) \]

For ease of discussion, artificially assign phases to the 3 growing BECs.

(There are really **only two independent phase variables**.)

Conditions for circular flow
(all phase differences between 0 and \(2\pi\))

<table>
<thead>
<tr>
<th>Clockwise Circulation</th>
<th>Counter-Clockwise Circulation</th>
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<tbody>
<tr>
<td>(\Phi_3 - \Phi_2 &lt; \pi)</td>
<td>(\Phi_3 - \Phi_2 &gt; \pi)</td>
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<tr>
<td>(\Phi_2 - \Phi_1 &lt; \pi)</td>
<td>(\Phi_2 - \Phi_1 &gt; \pi)</td>
</tr>
<tr>
<td>(\Phi_1 - \Phi_3 &lt; \pi)</td>
<td>(\Phi_1 - \Phi_3 &gt; \pi)</td>
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denotes relative phase

Given random relative phases, vortex nucleation might occur up to 25% of the time.

"Ideal" phases:
- \(\Phi_1=0, \Phi_2=2\pi/3, \Phi_3=4\pi/3\)
- \(\Phi_1=0, \Phi_2=4\pi/3, \Phi_3=2\pi/3\)
To model the growth of 3 independently-formed seed condensates:

**GPE**
*T=0, quasi-2D, split-step Fourier transform method*

**Initial conditions**
- Generate 3 independent condensates in segmented trap
- fixed number of atoms
- scattering length \( a_s = 0 \)
- assign phase differences

**Growth**
- Increase \( a_s \) (or increase atom number)

**Main Variables**
- Rate of increase of \( a_s \)
- Initial phase differences (random in experiment)
- Barrier height, shape
3 seed BECs are generated. Phases assigned.

Condensates grow in number, size

Atoms flow over barriers

A vortex core is formed and pinned at the central barrier
Does a vortex always form?

**No.** Given *random* relative phase differences, about 25% of the time (depends on growth rate) will the relative phases be appropriate to establish a vortex, though not always on-center. However...

...a vortex core that forms off-center can drift towards the center and be pinned at the barrier.

Even when the barrier height is too low to poke a hole in the BEC, the barrier preferentially displaces atoms from the center. The vortex core is pulled to the center to displace the least number of atoms.
Simulation movies

$2\pi/3$ relative phases (ideal case) faster formation, with non-ideal phases (note “turbulence vortices”)}
0,2π/3,4π/3 phase split,
5.0 s BEC growth

0,2π/3,4π/3 phase split,
1.0 s BEC growth
\[ 0,2\pi/3, (0.8)^*4\pi/3 \]
phase split,

1.0 s BEC growth
Angular momentum

Time dependence of angular momentum per atom, varying time to increase $a_s$. 3 seconds of dynamics for each case.

\[
\frac{1}{N\hbar} \left\langle \hat{L}_z \right\rangle = \hat{L}_z = -i\hbar \left[ x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right]
\]

Slower = more angular momentum, and fewer “turbulence vortices”.
Experiment: Optical Barrier and BEC imaging

Maskless Lithography Tool
Prof. Tom Milster
College of Optical Sciences

(1) Direct Imaging
- Chrome transmission mask

(2) Computer-Generated Hologram
- Phase holograms etched from photoresist
Experiment sequence:

- TOP trap strength: Relax to a weak TOP trap, then free expansion.
- RF cooling ramp: Cool to BEC.
- Barrier height: Turn on barrier, ramp down barrier.
- Image acquisition: Absorption image.

Final state of BEC: depends on Barrier Height.
Condensation in the presence of the barriers

**Experimental data:** In-trap, phase contrast images of fully formed BECs.

Increasing power in the optical beam

- **No Barriers.**
  - TF radius ~ 30 μm

- **Barrier beam**

- **Ramp down medium-power barriers:** look for vortices due to BEC growth

- **Ramp down high-power barriers:** look for vortices due to ramp-down

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190 μm

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Experimental Results

In-trap, phase-contrast image (with barrier on)

Absorption images after ballistic expansion
Results summary

**Experimental conditions**
- 10 second final RF ramp to create BEC.
- Medium-intensity optical barrier (a final merged BEC).
- 100-500 ms ramp down of optical barrier

**Results**

Observation of at least one vortex core: ~40%

>25%: turbulence, BEC growth rate is probably important

Add up to 1 s. before barrier ramp down, vortex observation probability drops to 0%.

**Vortices form during BEC growth**, not during barrier ramp

*Instead, use high-intensity barriers so that 3 final BECs form:*
- up to 60% probability of vortex observation
- **Vortices form during ramp**
- Vortex observation probability unaffected by extra time before ramp down
- multiple vortices often seen
- faster ramp = more vortex cores
- short (<100 ms) vortex lifetime
Image Gallery
A more basic experiment sequence

- Relax to a weak TOP trap
- Turn on barrier
- Cool to BEC
- Free expansion

TOP trap strength
- Relax to a weak TOP trap
- Free expansion

RF cooling ramp
- Cool to BEC
- Ramp down barrier

Barrier height
- Turn on barrier
- Ramp down barrier

Image acquisition
- Absorption image

Time:
- 10 s
- 0.5 s
- 0.05 s
Spontaneous Formation of Vortices by Evaporative Cooling

A single vortex observed **up to 10%** of the time, just by evaporative cooling in 3D trap (optical barrier beam is absent)!


- Kibble, J Phys A 9, 1387(1976),
- Zurek, Nature 317, 505 (1985),
- Anglin and Zurek, PRL 83,1707 (1999)

Spontaneous formation of vortices in BEC during evaporative cooling, also predicted by:

- Marshal, New, Burnett, and Choi, PRA 59, 2085 (1999),
- Drummond and Corney, PRA 60, R2661 (1999)
Conclusions

• When BECs merge and interfere, turbulence and vortices may result. Can happen by intentionally merging BECs, or by condensing in a bumpy potential.

• Vortices may be used as tools for examining fragmentation, phase dynamics.

• Further work may aid in studying superfluids in more “dirty” systems (e.g., random defects in superconducting systems), and in disordered systems.

• Direct phase imprinting of a split BEC might be used to controllably create vortex states.

• Vortices can spontaneously form during evaporative cooling. Spontaneous symmetry breaking during a temperature quench?

Next Steps:

• Vary time scales for BEC growth in both smooth and bumpy potentials
• Quasi-2D geometry (optical trap)
• Add more roughness to potential well
• Better examination of early stages of BEC formation?
... continuing an old Tucson tradition of vortex research